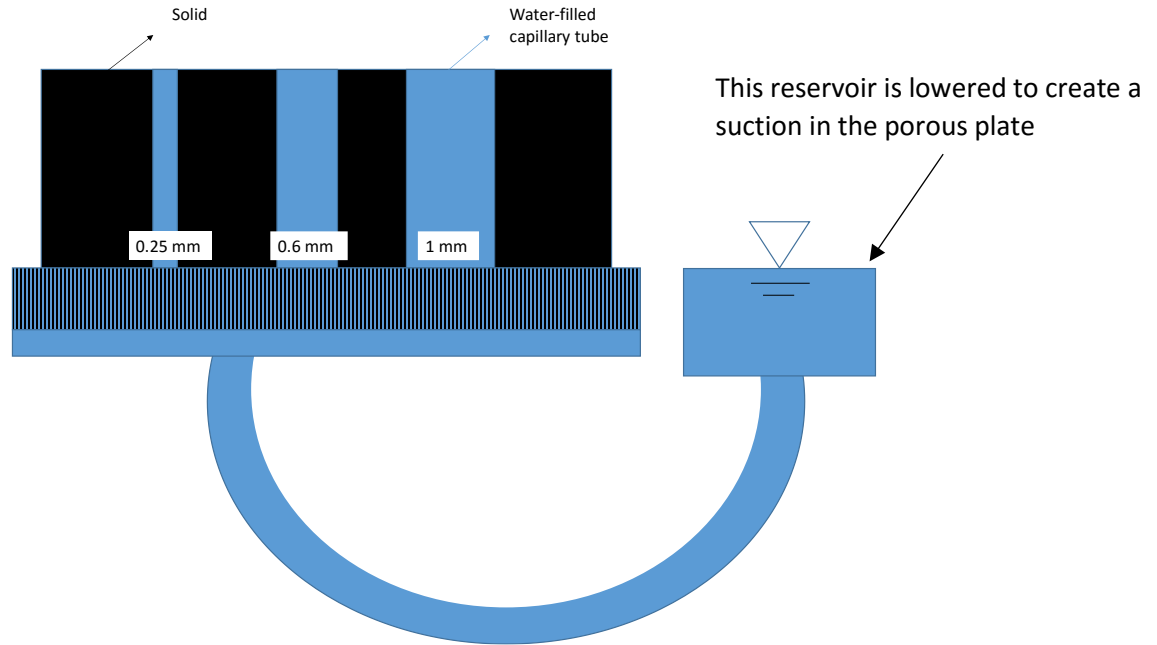




The above figure is an idealized “soil”. It consists of a solid block (0.5 cm x 0.5 cm x 0.5 cm) with three capillaries (radii shown in the figure).

The block is placed on a porous plate (the plate has pores with radii < 0.25 mm).

The plate is connected to a water reservoir connected to a tube/reservoir that allows the height of water to be adjusted. Lowering the reservoir applies a suction to the “soil” and removes water from the capillaries.

Initially, the height of water is level with the solid block, and the capillaries are filled with water.

By lowering the reservoir, a suction is applied to the “soil”. The value of the suction is the distance that the reservoir is lowered.

Question 1

- (i) Why is the pore size radius in the porous plate smaller than 0.25 mm?
- (ii) Plot the soil moisture characteristic curve (i.e., h vs θ or θ vs h , or use $\Psi = -h$ in place of h , as you wish) for this “soil” – use Jurin’s law.
- (iii) Does the curve in (ii) exhibit hysteresis? Briefly explain.

Question 2

In Question 1, the capillary tubes were initially full, and then drained by lowering the reservoir level. In this question, water is added to the top of the “soil” as a thin film (exposed to the atmosphere, so $p_{\text{water}} = 1 \text{ atm}$, or $h_{\text{water}} = 0$). In this situation, water will continuously flow downwards through the capillary tubes, pass through the tube, and exit at the reservoir. Assume that the flow in each capillary stops when air enters, i.e., when the suction applied at the base of the tube reaches the tube’s capillary pressure head value (as just calculated in Question 1).

Flow in each tube is given by Poiseuille’s law, which in terms of pressure head (gravitational head is ignored in this example), is:

$$Q = \rho g \pi r^4 (\Psi/L) / (8\mu)$$

where Q is the volume flux (i.e., volume per unit time, positive downwards), r is the tube radius, and L is its length (5 mm) (other symbols have their usual meaning).

- (i) How does the total volume flux through this “soil” vary with Ψ ? Here, plot the total flux as it varies with Ψ .
- (ii) Calculate the unsaturated hydraulic conductivity of this “soil” as it varies with Ψ .

Answers to Questions 1 and 2 are on the next page

Answer 1

- (i) The purpose of the porous plate is to provide a suction to the “soil”. If the pore size of the plate is larger than the pore size of the “soil”, when the reservoir is lowered, the porous plate will drain as well as the largest pore, then air will enter into the water below the plate. When this happens, air sits above the water (just below the bottom of the plate), and the water in the “soil” is disconnected from the reservoir.
- (ii) Jurin’s law applies in each tube. The contact angle between the water and tube is 0, and Jurin’s law is:

$$l = 2\sigma / (r\rho g)$$

where l is the height of water in the tube, σ is the surface tension, and r is the tube radius. Recall that, here, the height of water is a measure of capillary pressure ($p_c/\rho g$) – l is the suction applied by the soil to raise the water up the tube, so it is also the suction that must be applied to remove the water from the tube. We apply this suction by lowering the reservoir by the same distance.

Values:

$$\sigma = 72 \times 10^{-3} \text{ N/m} = 72 \times 10^{-3} \text{ kg/s}^2$$

$$g = 9.81 \text{ m/s}^2$$

$$\rho = 10^3 \text{ kg/m}^3$$

$$l(r = 0.25 \times 10^{-3} \text{ m}) = 5.87 \text{ cm}$$

$$l(r = 0.6 \times 10^{-3} \text{ m}) = 2.45 \text{ cm}$$

$$l(r = 1 \times 10^{-3} \text{ m}) = 1.47 \text{ cm}$$

$$\text{The total “soil” volume is } V_T = 0.5^3 = 0.125 \text{ cm}^3$$

$$\text{The total void space is } V_v = \pi \times 0.025^2 \times 0.5 + \pi \times 0.06^2 \times 0.5 + \pi \times 0.1^2 \times 0.5 = 0.022 \text{ cm}^3$$

The porosity is $V_v/V_T = 0.18$ – this is the saturated moisture content, θ_s .

As the reservoir is lowered, the tubes empty successively, starting from the largest r . The moisture contents for the “soil” are:

First, emptying the $r = 1 \text{ mm}$ tube, $\theta = 0.053$

Second, emptying the $r = 0.6 \text{ mm}$ tube, $\theta = 0.0078$

Third, emptying the $r = 0.1 \text{ mm}$ tube, $\theta = 0$

As the reservoir moves downwards, suction increases and the water drains from the “soil”. The points that can be plotted are written as the pairs (distance reservoir is lowered = suction = Ψ , moisture content = θ). These pairs are:

(Ψ , θ)

(0, 0.18)

(1.47, 0.053)

(2.45, 0.0078)

(5.87, 0)

Between the plotted points, the question is how the capillaries drain with increasing suction. If the drainage is linear with suction, then θ will vary linearly also.

- (iii) Little to no hysteresis – the tubes are straight, so the main cause of hysteresis (non-uniform pore sizes) is not present.

Answer 2

Note that this question has a major difference with the first. Here, water is available at the surface and flows through a given capillary until the pore drains. Below, we assume that each capillary drains suddenly.

The given formula is a version of Poiseuille's law describing the flow in each capillary tube. Flow takes place until the suction at the base exceeds the tube's capillary pressure head, and the tube empties.

When the suction (Ψ) applied by the reservoir is in the range (0, 1.47 cm), all the capillaries are full. The total flux, Q_T , is:

$$Q_T = (\sum r_i^4) \times \rho g \pi (\Psi/L) / (8\mu), \text{ where } r_1 = 0.25 \text{ mm}, r_2 = 0.6 \text{ mm and } r_3 = 1 \text{ mm}.$$

When the suction at the base of the "soil" exceeds 1.47 cm, the large tube ($r_3 = 1 \text{ mm}$) empties, and does not contribute to the flow, i.e., for Ψ in the range (1.47 cm, 2.45 cm), the total flux is:

$$Q_T = (\sum r_i^4) \times \rho g \pi (\Psi/L) / (8\mu), \text{ where } r_1 = 0.25 \text{ mm}, r_2 = 0.6 \text{ mm}$$

Similarly, when the suction is in the range (2.45 cm, 5.87 cm), only the smallest tube participates in the flow, so

$$Q_T = r_1^4 \times \rho g \pi (\Psi/L) / (8\mu), \text{ where } r_1 = 0.25 \text{ mm}$$

For a suction greater than 5.87 cm, all the tubes are empty, i.e., there is no flow, so $Q_T = 0$.

The plot (exercise left for you) of Q_T vs Ψ will consist of steps (at the transition where the tube suddenly stops participating). Between the steps, Q_T varies linearly with the suction, as given in the above equation.

- (ii)

The "soil" is subjected to increasing suction. As the suction increases, the tubes successively stop contributing to the flow. This is an analogy to a real soil where, with increasing suction applied to the soil, larger pores successively drain, and do not contribute to the flow. To make the analogy with porous media flow, recall that the Darcy flux is $q = Q_T/A$. The above formula for Q_T can be written as a constant (the "hydraulic conductivity"/A) times a gradient. Specifically,

$$\text{"hydraulic conductivity"} = (\sum r_i^4) \times \rho g \pi / (8\mu)$$

$$\text{gradient} = (\Psi/L)$$

and A is cross-sectional area, $A = 0.25 \text{ cm}^2$ in this case.

To get the above formula for “hydraulic conductivity” as a function of Ψ (the suction) instead of r , use Jurin’s law (recognising that l in Jurin’s law is the same as Ψ). The plot will be a series of steps with abrupt changes (as the different capillary tubes suddenly drain).